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AN UNSTEADY PROBLEM OF FRICTIONAL CONTACT FOR A CYLINDER WITH ALLOWANCE FOR HEAT RELEASE AND WEAR

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Mechanical interaction of machine elements is inevitably accompanied by friction and wear. Friction and wear are fundamental aspects of ensuring high performance, reliability, and durability of machines and mechanisms. Instability of contact parameters (pressure, temperature, thermoelastic deformations, wear, etc.), which disturbs the normal operation of tribocontacts, is a very important problem.

The aim of the present paper is to construct and analyze an unsteady solution of the problem of thermoelastic contact of a cylinder with a rigid yoke under conditions of frictional heating and abrasive wear and also to study the characteristic features of the model, in particular, thermoelastic instability (TEI). Goryacheva and Dobychin [1] analyzed the influence of wear coefficients of various types on the value of wear using this model problem and ignoring heat production. Morov [2] the studied TEI in structures such as radial compactors in the absence of wear. Grilitskii and Kul'chitskii-Zhigailo [3] studied the features of thermoelastic contact using as an example two contacting cylinders with their relative rotation. Problems of frictional contact have been usually studied for specified compressive forces in a steady-state formulation [4, 5].

We assume that the contact region and geometry of the contacting bodies are such that a one-dimensional model can be used. Use of one-dimensional models is justified by the possibility of investigating the typical features inherent in actual friction assemblies [1, 5, 6].

1. Formulation of the Problem. An elastic heat-conducting cylinder of radius r_0 is inserted into a rigid yoke with tension u_0 (Fig. 1). The cylinder is rotated with a constant angular velocity ω around the z axis. A frictional force $F = 2\pi r_0 f p$ arises in the region of contact between the cylinder and the yoke, resulting in heat production and wear of the cylinder surface. Let us assume the abrasive-wear law according to [1, 7]. The heat transfer between the cylinder and the yoke obeys Newton's law.

In our case, in which the cylinder displacement along the z axis equals zero and the displacements u_r are only functions of time t and radial coordinate r, the temperature $\theta(r,t)$, the contact pressure $p(t)$, and the cylinder wear $u_w(t)$ are to be found.

To solve this problem, one must integrate the system of differential equations of quasi-static unrelated thermoelasticity [8],

$$\frac{\partial^2}{\partial r^2} u_r(r,t) + \frac{1}{r}\frac{\partial}{\partial r} u_r(r,t) - \frac{1}{r^2} u_r(r,t) = \alpha \frac{1+\nu}{1-\nu}\frac{\partial}{\partial r}\theta(r,t),$$
$$\frac{\partial^2}{\partial r^2}\theta(r,t) + \frac{1}{r}\frac{\partial}{\partial r}\theta(r,t) = k^{-1}\frac{\partial}{\partial t}\theta(r,t), \quad r \in (0, r_0), \quad t \in (0, t_c), \tag{1.1}$$

subject to the mechanical conditions

$$u_r(0,t) = 0, \quad u_r(r_0,t) = -u_0 + u_w(t), \quad t \in (0, t_c), \tag{1.2}$$

the thermal conditions

$$2\pi r\lambda \frac{\partial}{\partial r}\theta(r,t) \to 0, \quad r \to 0, \quad \lambda \frac{\partial}{\partial r}\theta(r_0,t) + \alpha_\theta \theta(r_0,t) = f\omega r_0 p(t), \quad t \in (0, t_c), \tag{1.3}$$

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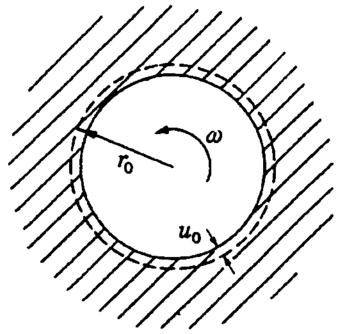

Fig. 1

and the initial conditions

$$\theta(r,0) = 0, \quad r \in (0, r_0). \tag{1.4}$$

The cylinder wear is proportional to the work of frictional forces [1, 7]:

$$u_w(t) = K_w \omega r_0 \int_0^t p(\eta) d\eta, \quad 0 < t < t_c. \tag{1.5}$$

The radial stresses for the cylinder are found from the formula

$$\sigma_r = \frac{E}{1-2\nu}\left[\frac{1-\nu}{1+\nu}\frac{\partial}{\partial r}u_r + \frac{\nu}{1+\nu}\frac{1}{r}u_r - \alpha\theta\right].$$

In (1.1)–(1.5), E is the Young modulus; ν, λ, k, α, f, α_θ, and K_w are the Poisson's ratio, heat conductivity, thermal diffusivity, temperature-expansion factor, friction, heat-transfer coefficient, and wear rate of the cylinder material, respectively; and $p(t) = -\sigma_r(r_0, t)$ is the contact pressure. The time of contact t_c is determined as the time at which the contact pressure is nonnegative, i.e., $p(t) \geqslant 0$ at $t \in (0, t_c)$.

2. Representation of the Solution. Let us introduce dimensionless quantities

$$R = r/r_0, \quad \tau = t/t_*, \quad \Omega = 2E\alpha k/[\lambda(1-2\nu)], \quad v = \omega f \Omega/\omega_*, \quad \tau_c = t_c/t_*,$$
$$\mathrm{Bi} = \alpha_\theta r_0/\lambda, \quad \xi = K_w E_1/f\Omega, \quad E_1 = E/[(1+\nu)(1-2\nu)]$$

and reference parameters

$$t_* = \omega_*^{-1} = r_0/k^2, \quad p_* = E_1 u_0/r_0, \quad \theta_* = u_0/[2\alpha(1+\nu)r_0].$$

Using the Laplace integral transform [9], we can write the solution of boundary-value problem (1.1)–(1.5) as

$$\theta(R,\tau) = \theta_* v \sum_{m=1}^{\infty} \frac{\Delta_3(R, s_m)}{\Delta'(s_m)} \exp(s_m\tau), \quad p(\tau) = p_* v \sum_{m=1}^{\infty} \frac{\Delta_2(s_m)}{s_m \Delta'(s_m)} \exp(s_m \tau),$$
$$u_w(\tau) = u_0\left[1 + v\sum_{m=1}^{\infty}\frac{\xi\Delta_1(s_m)}{s_m\Delta'(s_m)}\exp(s_m\tau)\right], \tag{2.1}$$

where

$$\Delta'(s_m) = \frac{d}{ds}\Delta(s)\Big|_{s=s_m} = 0.5\big\{D_m[(\mathrm{Bi}+2)s_m + v\xi\mathrm{Bi}] + C_m[2\mathrm{Bi} + s_m + v(\xi - 1)]\big\};$$
$$\Delta_1(s_m) = \mathrm{Bi}\, C_m + s_m D_m; \quad \Delta_3(R, s_m) = I_0(R\sqrt{s_m}); \quad \Delta_2(s_m) = s_m D_m - \xi\Delta_1(s_m);$$
$$\Delta(s) = s\Delta_1(s) - v\Delta_2(s); \quad D_m = I_1(\sqrt{s_m})/\sqrt{s_m}; \quad C_m = I_0(\sqrt{s_m});$$

$m = 1$ and 2; $I_n(x)$ are modified nth-order Bessel functions of the first kind; and s_m are roots of the characteristic equation $\Delta(s) = 0$ $(m = 1, 2, \ldots)$. Studies show that normally $\mathrm{Im}\, s_m = 0$ for $m = 3, 4, \ldots$.

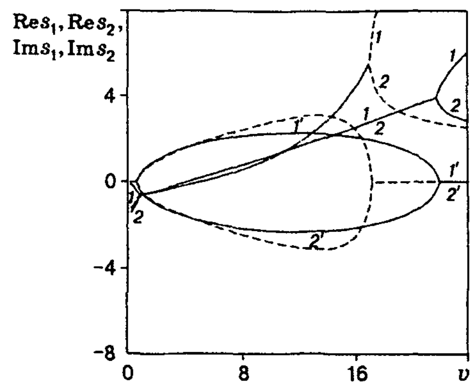

Fig. 2

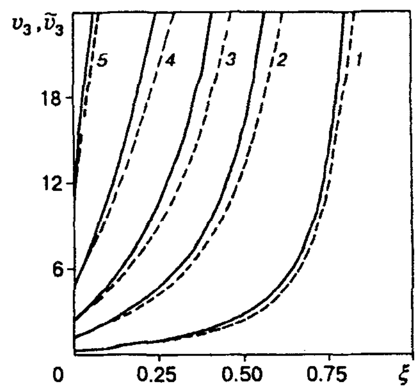

Fig. 3

For $m = 1$ and 2, the roots lie, depending on the problem parameters, in the right-hand or left-hand complex half-plane s.

If $\xi < \xi_1$ [$\xi_1 = 1/(1 + \text{Bi}/2)$], the roots are negative for $v < v_2$, complex-conjugate with a negative real part for $v_2 < v < v_1$, complex-conjugate with a positive real part for $v_1 < v < v_3$, and positive for $v_3 < v$. Curves of the real and imaginary parts of the roots s_1 and s_2 are shown in Fig. 2 by solid curves versus the dimensionless velocity v ($\xi = 0.4$ and Bi $= 1$). Curves 1 and 1′ (2 and 2′) correspond to the real and imaginary parts of the root s_1 (s_2).

For $\xi > \xi_1$, the roots s_1 and s_2 always lie in the left-hand complex half-plane s. They are negative for $v < v_2$, complex-conjugate for $v_2 < v < v_3$, again negative for $v_3 < v < v_4$, and, for $v_4 < v$, the roots s_2 and s_3 are complex-conjugate and the root s_1 is negative. Thus, for $v = v_m$ ($m = \overline{1,4}$) the properties of the roots of the characteristic equation change.

The velocity v_3 versus the parameter ξ for different Biot numbers is plotted by the solid curves in Fig. 3. Curves 1–5 correspond to Bi $=$ 0.1, 0.5, 1.0, 2.0, and 5.0.

The value of $\pi/\text{Im}\, s_1$ can be correlated with the contact time τ_c: the larger $\text{Im}\, s_1$, the shorter the contact time. The quantity $\text{Re}\, s_1 > 0$ reflects the increase in the contact characteristics and their extreme values.

Expanding the function $\Delta(s)$ into a power series in the neighborhood of zero, for small values of s, we can write the roots s_1 and s_2 as

$$s_{1,2} = \frac{v(1 - \xi/\xi_1) - v_0 \pm \sqrt{v^2(1-\xi)^2 - 2vv_0(1 + \xi/\xi_1) + v_0^2}}{2/\xi_1 - v(1 - \xi/\xi_2)/4}, \quad \xi_2 = (1 + \text{Bi}\,/4)^{-1}, \; v_0 = 2\text{Bi}. \tag{2.2}$$

The changes in the real and imaginary parts of approximation of the roots s_1 and s_2 versus the dimensionless velocity v are shown in Fig. 2 ($\xi = 0.4$ and Bi $= 1$) by dashed curves.

Relations (2.2) allow one to write approximate expressions for v_m, $m = 1, 2, 3$. Thus, $v_m \approx \tilde{v}_m$, $m = 1$, 2, 3, where

$$\tilde{v}_1 = \frac{v_0 \xi_1}{\xi_1 - \xi}, \quad \tilde{v}_m = v_0 \frac{1 + \xi/\xi_1 \mp 2\sqrt{\xi(1 + \xi \text{Bi}/4)/\xi_2}}{(1-\xi)^2}, \quad m = 2, 3.$$

The dimensionless velocity $\tilde{v}_3$ versus the parameter ξ for different Bi values is shown in Fig. 3 by dashed curves. Curves 1–5 correspond to Bi $=$ 0.1, 0.5, 1.0, 2.0, and 5.0.

3. Analysis of the Solution. Taking into account the properties [9] of the Laplace transformants of the solution, we obtained asymptotics of characteristics of thermoelastic contact for the initial time:

$$\theta(1,\tau)/\theta_* = v2\sqrt{\tau/\pi} + O(\tau^{1.5}), \quad p(\tau)/p_* = 1 + v(1-\xi)\tau + O(\tau^2), \quad u_w(\tau)/u_0 = v\xi\tau + O(\tau^2).$$

We analyze special cases. In the absence of convective heat transfer (Bi $= 0$), the contact pressure and

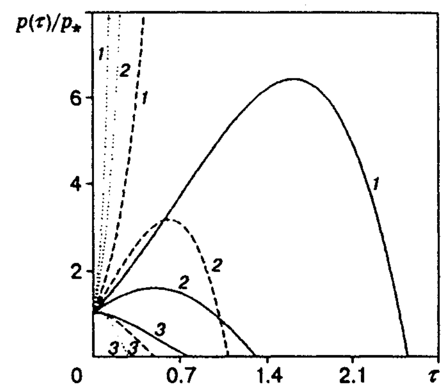

Fig. 4

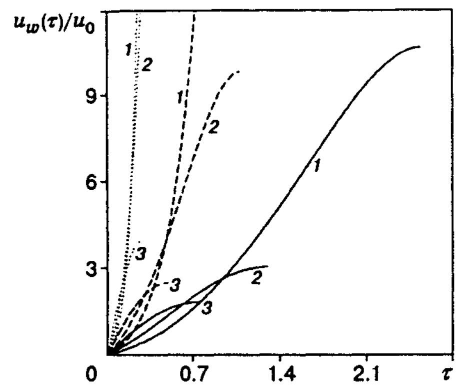

Fig. 5

wear for an arbitrary time take a simple form:

$$p(\tau)/p_* = \exp[-v(\xi - 1)\tau], \quad u_w(\tau)/u_0 = \xi\big\{1 - \exp[-v(\xi - 1)\tau]\big\}/(\xi - 1);$$

for ideal convective heat transfer (Bi $= \infty$),

$$p(\tau)/p_* = \exp(-\beta\tau), \quad u_w(\tau)/u_0 = 1 - \exp(-\beta\tau), \quad \beta = v\xi.$$

The contact characteristics have the same form at small velocities of relative motion if heat production is ignored ($f \to 0$).

Let us study this problem by analyzing solutions (2.1) and the behavior of the roots of the characteristic equation. The parameter $\xi = \lambda K_w / 2 f \alpha k (1 + \nu)$ describes the hierarchy of wear and thermal expansion.

If there is no wear ($\xi = 0$) and the velocity v is smaller than the critical velocity v_0, the contact pressure and temperature enter a steady-state regime,

$$p_c = p_* v_0 / (v_0 - v), \quad \theta_c(R) = \theta_* 2v / (v_0 - v),$$

since the heat generation and its removal are mutually compensated in the system. If the velocity v is higher than the critical velocity v_0, the temperature and contact pressure grow exponentially. The system does not cool, and frictional TEI arises, i.e., a minor external excitation of the system (in our case, compression of the rotating cylinder) causes an exponential increase in temperature and contact pressure.

For $0 < \xi < \xi_1$, i.e., when the thermal expansion exceeds the wear and $v \leqslant v_2$, the contact time $\tau_c = \infty$ and the contact characteristics tend to their steady-state values: $p_c = 0$, $\theta_c(R) = 0$, and $u_w = u_0$. The closer v to v_2, the longer the time required for the attainment of the steady-state regime. In the range of $v_2 < v < v_3$, the contact time is limited. The minimum contact time is observed for velocities $v \approx (v_2 + v_3)/2$, i.e., when $\mathrm{Im} s_1$ acquires a maximum value. When the velocity v approaches v_3, the peak values of the contact characteristics increase. If the velocity v is higher than the critical velocity v_3 (the region above the corresponding curves in Fig. 3), frictional TEI is observed, i.e., the contact characteristics grow exponentially as $\exp(s_1 \tau)$.

For $\xi \geqslant \xi_1$, i.e., when the wear is larger than the thermal expansion and $v \leqslant v_2$, the contact characteristics tend in time to a steady-state solution of the problem. For $v \geqslant v_2$ (Bi > 0), the contact time τ_c is limited although a steady-state solution formally exists. As the slip velocity increases, the contact time decreases.

The contact pressure for $\xi \geqslant 1$ always tends monotonically to zero, in contrast to the case $0 < \xi < 1$, in which it has a maximum in the absence of thermoelastic instability.

4. Numerical Results. To illustrate the theoretical studies of the behavior of the contact characteristics, we perform a numerical analysis of the solution of the problem for various values of the coefficient ξ, which describes the value of wear, and for various velocities v; Bi $= 1$ ($\xi_1 = 0.67$).

For a steel cylinder [$\alpha = (14 \cdot 10^{-6})°\mathrm{C}^{-1}$, $\lambda = 21$ W/(m $\cdot$ °C), $k = 5.9 \cdot 10^{-6}$ m^2/sec, $\nu = 0.3$, and

$E = 190 \cdot 10^9$ Pa] and for $r_0 = 3 \cdot 10^{-2}$ m and $u_0 = 1 \cdot 10^{-6}$ m, the normalization parameters are as follows: $t_* = 153$ sec, $p_* = 1.22 \cdot 10^7$ Pa, and $\theta_* = 0.92°$C.

Figures 4 and 5 show the contact pressure $p(\tau)/p_*$ and wear $u_w(\tau)/u_0$ versus the dimensionless time τ (the Fourier number). The solid, dashed, and dotted curves correspond to dimensionless velocities $v = 5$, 10, and 25, respectively. Curves 1–3 are plotted for $\xi = 0.2$, 0.4, and 0.8.

For $\xi = 0.2$, we have $v_3 = 8$, $v_2 = 0.9$, and $v_1 = 2.7$ ($\tilde{v}_3 = 7.3$, $\tilde{v}_2 = 0.9$, and $\tilde{v}_1 = 2.9$). TEI arises for $v > v_3$ (dashed and dotted curves).

For $\xi = 0.4$, we have $v_3 = 21.9$, $v_2 = 0.65$, and $v_1 = 4.1$ ($\tilde{v}_3 = 17.1$, $\tilde{v}_2 = 0.65$, and $\tilde{v}_1 = 5$). TEI is observed for $v > v_3$ (dotted curves). With an increase in the velocity, the contact time decreases, the peak values of the contact pressure and temperature increase, and the wear also increases.

For $\xi = 0.8$, there is no TEI for all values of the velocity v. With an increase in the velocity, the contact time is reduced, and the peak value of the contact temperature and the wear increase.

For a fixed velocity, an increase in the wear rate ξ leads to a smaller contact time, a more intense increase in wear in the initial stage, and also to a smaller final value of wear and a smaller contact time.

Let us sum up the results.

1. An explicit solution of a spatially one-dimensional problem of frictional contact was constructed and analyzed.

2. Conditions for the occurrence of frictional thermoelastic instability were determined: Bi $\in [0, \infty)$ and $\xi \in [0, \xi_1)$, $v \in [v_3, \infty)$.

3. Allowance for wear leads to a higher critical velocity v_3 at which thermoelastic instability arises, and for $\xi > \xi_1$, i.e., when wear dominates over thermoelastic expansion, frictional TEI disappears altogether. Thus, for the given model the wear is a stabilizing factor, and this is confirmed by the results of [2].

4. For the given model of frictional contact, TEI arises not only when the roots of the characteristic equation are located in the right-hand complex half-plane of the Laplace-transform parameter s ($v > v_1$ and $\xi < \xi_1$) but also when they have a zero imaginary part ($v \geqslant v_3$). The latter condition distinguishes the obtained conditions from the commonly accepted conditions.

5. Analytical expressions for the characteristic velocities were determined, which allows one to predict the behavior of the frictional contact characteristics in time.

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